

Global disk model for galaxies NGC 1365, NGC 6946, NGC 7793, UGC 6446

Joanna Jałocha¹, Łukasz Bratek¹, Marek Kutschera^{1,2} and Piotr Skindzier²

¹*Institute of Nuclear Physics, Polish Academy of Sciences, Radzikowskiego 152, PL-31342 Kraków, Poland*

²*Institute of Physics, Jagellonian University, Reymonta 4, PL-30059 Kraków, Poland*

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ABSTRACT

We study spiral galaxies using a simple global disk model as a mean for approximate determination of mass profiles. Based on rotation curves and the amount of gas (Hi+He), we find global surface mass densities consistent with the measurements and compare them with B-band surface brightness profiles. As a result we obtain mass-to-light ratio profiles. We give some arguments for why our approach is reliable and sometimes better than those assuming ad hoc the presence of massive non-baryonic dark matter halo. With this model, we study galaxies NGC 7793, NGC 1365, NGC 6946 and UGC 6446. Based on THINGS- rotation curve we study also galaxy NGC 4536 and compare the results with those we published elsewhere for the same galaxy.

Key words:

1 INTRODUCTION

Reliable determination of total mass of spiral galaxies is a crucial step in the search for non-baryonic dark matter. In particular, laboratory experiments aimed at detection of dark matter particles, could succeed only if there is sufficient amount of dark matter inside the disk of our Galaxy in the Sun's vicinity.

Present models of mass distribution in spiral galaxies assume that in addition to stellar disk, bulge, luminous halo, there is massive halo of non-baryonic dark matter, most often assumed spherical. It dominates the total galaxy mass and also governs the dynamics of outer parts of the stellar disk. This last feature makes it possible to test in some cases, if spherically symmetric rather than flattened dark matter distribution is actually present.

Recently, we have analyzed mass distribution in several spiral galaxies (Jałocha et al. 2007, 2008; Bratek et al. 2008). We find that for some galaxies it cannot be spherical at larger radii. Since we expect in this case a flattened mass distribution to better approximate the gravitational potential at larger radii than the spherical one, we apply the global disk model. We then find that mass distribution of luminous matter accounts for rotation curves of the examined galaxies.

There is also increasing number of studies whose authors, using more involved galaxy models than so far, conclude, contrary to earlier findings, that luminous matter accounts for rotation in the internal galactic regions (Sellwood & Kosowsky 2000; Evans 2001; Palunas & Williams 2000; Williams, Bureau & Cappellari

2009). There is also possibility that rotation of galaxies in the outermost regions can be driven by magnetic fields, not by dark matter (Battaner, Garrido, Membrado & Florido 1992).

These results need to be seen properly in the context of non-baryonic dark matter searches. Currently, there is strong observational evidence of non-baryonic dark matter in clusters of galaxies, in particular, from gravitational lensing by clusters and from colliding clusters, as e.g. the Bullet cluster (Clowe et al. 2006). Also, the cosmological model provides support to the non-baryonic dark matter hypothesis, albeit at a more theoretical level. On the other hand, direct searches give null results, except for DAMA/LIBRA (Bernabei et al. 2007), however, these results are disputable and have not been confirmed by other searches.

The consistent picture might emerge if one assumed that clustering scale of non-baryonic dark matter is that of galaxy clusters. At smaller scales, non-baryonic dark matter could be dilute. In particular, its contribution to spiral galaxy masses could be much less than anticipated in spherical halo models. It cannot also be excluded that spiral galaxies may differ from each other in abundance of dark matter – some of them could be rich in dark matter, the other might be devoid of it.

Before presenting results for some spiral galaxies obtained with our model based on a global disk approximation, we briefly signalize some important issues concerning rotation curve modeling.

The methods for ascertaining mass distribution in spiral galaxies commonly use very simplified, one would even say naive, one- or many-component galaxy models, that

assume symmetries of mass distribution (axial, spherical), properties of rotation fields (concentric circular), and composition characteristics (constant M/L ratios, *etc*), which at most approximately agree with observations or sometimes even contradict them. Our model of spiral galaxies is also simplified, however, it possess some features which, according to us, gives it the advantage over other models – e.g. it does not assume in advance any M/L ratio or its constancy, but is obtained at output as a resulting M/L ratio profile, which seems more realistic.

In the past, the prevailing opinion was that one needs dark matter inside galaxies to account for their rotation and to stabilize their allegedly cold disks, whereas there is increasing number of papers modeling the same galaxies without dark matter. This is achieved by considering sufficiently complex and thus also more "flexible" models, for example, those assuming wide, not thin, stellar disks, *ect*. There is also too much freedom in choosing galaxy models, and there is no general agreement which models are closer to reality.

But, why to assume circular orbits, axial or spherical symmetry, constant mass-to-light ratios when this is not the case for real galaxies? As is pointed out by Bosma (1978), most of spirals are not differentially rotating axisymmetric disks with only tiny disturbances due to spiral arms. There are large scale deviations from the symmetry, often with very large amplitudes or even with warped HI layers (Bosma 1978).

The predictions of the simple galaxy models might further be modified if one preferred to reconstruct whole Doppler images rather than rotation curves, the more the latter are only some interpretations of Doppler images obtained not without additional assumptions. Going even further, what would happen if, instead of using the simplified models, one used a sort of templates for inferring matter distribution in real galaxies based on their Doppler images and the observed qualitative characteristics like the number of arms, the presence of a bar *etc*? Such templates, relating mass distribution and its qualitative shape features to simulated Doppler images could be catalogued by elaborating a large number of galaxy-like virialized stationary systems from n -body simulations. Wouldn't this approach be the right step toward realistic modeling of mass distribution in galaxies? To what extent could this approach change our understanding of spiral galaxies and current opinions about their composition?

1.1 Dark matter inside galaxies

The question how much dark matter is present in galaxy's interiors remains still open (Bosma 1999), (Sellwood 1998) Sellwood & Kosowsky (2000) point out that the better are the data used for constraining the properties of galaxy halos and the better is the quality of simulations, there is more difficulties with dark matter models on galaxy scales. Also Evans (2001) says that the case for CDM, having «a number of stubborn problems», is weak judged on the data from galactic scales only.

This is especially evident from the still growing number of examples of morphologically various galaxies whose rotation curves can be accounted for without dark matter hypothesis by using non-canonical, more realistic approach to mass modeling. For example, even though still assum-

ing the simplifying unrealistic axi-symmetry and that the radial mass profile should follow that of luminosity with constant M/L ratio, Palunas & Williams (2000) were able, under the strict maximum (thick) disk hypothesis, that is, not including a dark halo, to reproduce – with I-band M/L ratio of $(2.4 \pm 0.9) \cdot h_{75}$ and consistently with stellar population synthesis models – the overall structure of the optical rotation curves for most of galaxies in their sample of 74 spiral galaxies with various surface brightness profiles and rotation curve morphologies. They link the poorest fits with non-axisymmetric features, like bars or strong spiral arms, that influence the brightness profiles, the determination of inclination, major-axis position angle, *etc*. This situation, however, might be cured by considering more realistic models.

That the interiors of spiral galaxies cannot be dominated by dark matter is also suggested by dynamical arguments. As noted by Palunas & Williams (2000), if the average projected surface mass density of dark matter halo was greater than that of the optical disk, the common instabilities would be suppressed (Athanasoulas, Bosma & Papaioannou 1987) together with lopsided modes (Rix & Zaritsky 1995) appearing in many disk galaxies. In addition, as they point out, there are models that reproduce features of rotation curves within the optical disks on different scales and that the models rarely require dark matter halos (Kalnajs 1983; Kent 1986; Buchhorn 1992).

These interesting observations of absence of dark matter inside galaxies are best expressed by Williams, Bureau & Cappellari (2009). They speculate, having succeeded in fitting some galaxies without dark matter, whether Occam's Razor argument would not be applicable. Why to introduce something that is not needed? The authors also form less authoritative statements about dark matter presence in galaxies than others. What they merely say is that dark matter presence is only statistically significant.

The presence of a dark matter component that dominates the outer mass distribution in spiral galaxies, seems evident from their optical rotation curves which do not fall off sufficiently fast close to their optical edge (Rubin, Thonnard & Ford 1978) and from their extended HI rotation curves (Bosma 1978). However, in the context of what has been said above, one can also conjecture that the halo would disappear if the galaxy models were more realistic – allowing for changing mass-to-light ratios, assuming non-axisymmetric mass features (such as spiral arms and bars), if the assumed extinction models was altered *etc*. Such models should be also taking into account that the velocity field in galaxies is much more complicated than circular and that the published rotation curves are obtained by reducing a two-dimensional Doppler image of a galaxy into a single one-dimensional curve (such an image by itself carries already a reduced, partial information about rotation), and that this reduction is performed not without additional assumptions concerning the interpretation, averaging methods, *etc*, of the Doppler image (for example, an averaging method which produces rotation curve by calculating pairs $[r, \sqrt{(rv^2)/\bar{r}}]$ scanning a Doppler image, where the averaging at a given r is carried out over a window of size Δr and weighted by, say, a function of local intensity of the image

within the window, is presumably good for circular orbits of test bodies in a spherically symmetric gravitational potential, but not necessarily for a self-gravitating system which may be flattened).

As concerns dark matter in outer galaxy regions, there is also possibility, never taken into account in modeling of flattened galaxies, that rotation of slightly ionized gas at large radii may be driven also by magnetic fields whose strength may be comparable with that of gravitation of whole galaxy at these radii (Sánchez-Salcedo & Reyes-Ruiz 2004; Kutschera & Jalocha 2004), helping to lower (or even eliminate) the amount of dark matter needed in the outskirts of galaxies where rotation curves are usually flat (Battaner, Garrido, Membrado & Florido 1992).

It is true that realistic modeling of galaxy rotation is very challenging and might be very difficult to achieve, but, at the same time it is not clear whether the naive models used so far, lead to trustworthy results – has the opposite ever been rigorously proved?

1.2 Dark halo and stability arguments

In modeling of galactic optical discs, it is assumed they are highly flattened, axisymmetric and cold. There is a common opinion linked with stability argument of Ostriker & Peebles (1973), that for such disks to be stable within the optical radius, one necessarily needs the galaxies to be immersed within massive dark matter haloes.

This conclusion comes about as follows. As observed in numerical simulations, when random stellar motions are small compared to the streaming rotational motion in flattened galaxies, without such a halo a bar would form in one rotation period, and only afterward axial symmetry would be regained. During this stage, approaching stationary state, the disk would heat up and could not remain cold (this would not agree with the assumption that disks are cold). Therefore, cold disks must be immersed in (usually spherical) halos of sufficiently large mass.

There are various criteria known for deciding whether a virialized rotating flattened system could be stable. Ostriker & Peebles (1973) conclude that a spherical halo with halo-to-disk mass ratio of 1 to 2.5 and initial value of total rotational kinetic energy to total gravitational energy ratio of 0.14 ± 0.03 renders the disk stable (of course, the halo mass beyond the disk's outer radius has no effect and should not be taken into account). Their conclusions are supported by many computer simulations and also by linear stability analysis carried out by Toomre for Mestel disk (Toomre 1964). Note, however, that for nonlinear differential equations (and hydrodynamical description of galaxies leads to such equations) it is not a priori obvious that linear approximation appropriately describes solutions, the more that the gravitational potential also evolves along with matter, thus also the instability modes and the associated spectrum of eigenvalues. Sellwood (1983) criticized the criterion, as well as various other criteria, to be of little use. Nevertheless, he points out, they all lead to similar demands on mass distribution. As shown in (Sellwood 1983), based on Fall & Efstathiou (1980) models, one comes to the conclusion that, for stability, total spherical mass interior to the half-mass radius of the disc should be more than twice the disc mass in the same region. Hohl (1976)

and Sellwood (1980) find similarly, using their numerical models, that for stability of differentially rotating disks the ratio of spheroidal mass to that of the disk interior to the outer radius of the disk, should be approximately 2, while for rigid disks the mass could be smaller (Hohl 1976). This spheroidal-to-disk mass criterion is much more handy to use than the Ostriker-Peebles-like criteria in modeling galaxies, since mass functions of different galaxy components are much simpler to find.

It should be noted, however, that the assumption of cold disks is based on the belief that the local observation of small dispersion of velocities in solar neighborhood in our own galaxy can be extrapolated onto whole Galaxy and even onto all other flattened galaxies. Even if the assumption was true, there is an argument due to Kalnajs (1987) that the halo is not so efficient a stabilizer as a bulge alone could be. A massive halo would change rotation field of the bar in a way not measured in real galaxies (Palunas & Williams 2000). As for modest halo, it would be difficult to decide about its presence since its effect would be small (Kalnajs 1987). It is also known that random motions of stars of the disc near the center have stabilizing property as well as a bulge or a thick disk. Neglecting this effect could result in overestimating the bulge+halo mass required for global stability.

Sellwood (1983) surmises that most galaxies might have only moderate fractions of mass in the bulge+halo. As concerns the internal galaxy regions comprising the optical disk, the amount of dark matter is very poorly known (Palunas & Williams 2000). On the other hand, contrary to the common opinion that only spheroidal halos can help to stabilize cold galactic disks, bulges (Kalnajs 1987) or thick disks (Athanas & Sellwood 1987) are now considered to be more efficient stabilizers (Sackett 1996). What's more, the presence of dark matter in large amounts, dominating the disk mass, would simply suppress the instabilities required to form the observed structures such as bars and spiral arms (Athanasoulas, Bosma & Papaioannou 1987).

Because spherical halo mass outside the optical radius has no effect on disk stability, the facts mentioned above, reverse the disk stability argument by Ostriker & Peebles (1973), frequently used as a support for the need of a massive dark matter halo within the optical radius (Palunas & Williams 2000). Here, it is worth to add that Sellwood & Evans (2001) construct a model disk galaxy with an almost flat rotation curve which is stable without presence of dark matter.

1.2.1 stability arguments and mass models

In modeling of mass distribution in galaxies, provided one has indeed a model that properly describes a given galaxy, one in principle should check whether the found mass distribution is stable. If disks were indeed cold, then for stability the spherical mass component should be sufficiently large. It seems however, that for estimating of the mass distribution in galaxies, the stability issue is not so important for astronomers, the more that the galaxy models used in this respect are very simple, one could say even naive. Majority of them assume idealized substructures such as disks and bulges with simple parametric profiles, and assume symmetries that are not observed (spherical and axial). For example, the spherical mass to disk mass ratio in model used by

de Blok et al. (2008)(THINGS) is much less than 2, *e.g.*, for galaxies NGC 4736, NGC 3031. Despite the fact that a dark matter halo was included, the ratio would suggest that the found mass distribution is unstable as it does not meet the stability criteria mentioned previously, that is, the disk-to-halo mass is too large.

There is a big 'but', however. All mass models for flattened galaxies are merely some idealizations that do not take into account different processes that make real galaxies stable. The role of the models is rather to roughly estimate the gravitational potential from rotation curves assuming circular orbits and axial symmetry, and so on, and then to derive the corresponding mass distribution which one believes to some degree approximate the actual amount of matter in real, and otherwise stable galaxies. For example, maximal disk models give "effective" surface density which should be treated rather as a substitution for different galaxy components both those flattened and spheroidal, described altogether by a single function of radius; only after a suitable de-projection one would infer about various galaxy components, thus it would be at least exaggeration to apply to such models the stability criteria which says that axisymmetric cool disks are unstable.

Of course, in real galaxies, masses are neither distributed in infinitely thin disks nor in ideally spherically symmetric structures, orbits are not circular, rotation field has a very complicated structure, and mass distribution does not have axial symmetry – axial symmetry is clearly broken by bars and spiral structure. Thus it would be at least inappropriate to qualify or disqualify different models on the basis of stability arguments even if they do not contain a halo of dark matter, since the models, by construction, do not describe real galaxies and ignore many subtleties. The models are only slightly more accurate than would be a dimensional analysis, which states that the amount of electromagnetic energy radiated by a galaxy should be given by $L = \alpha D_c V_c^2 / (G\mu)$, where V_c is some characteristic velocity derived from Doppler image, D_c is its radial size estimated based on the spatial range of the luminous stuff, μ is some mass-to-light ratio derived on population analysis ground, and α is some unknown dimensionless factor related to the particular geometry of mass distribution and of rotation field. All parameters apart from α can be estimated from measurements. The models only better approximate the parameter α by giving its concrete model, more or less accurate. For example, the estimation should be better when for an elliptical galaxy one uses a spherical rather than a disk-like model, or, for a spiral galaxy, a disk model rather than spherical. The approximation should improve when one distinguishes spherical and disk-like subcomponents, further, one can add a width to the disk, and so on. But it would be hard to treat these models seriously when dynamics and stability of a galaxy is concerned (especially when maximal disk model is used which treats on equal footing flat and spheroidal internal mass components).

Without realistic modeling such difficult problems as structural stability, cannot be answered. By realistic modeling we understand finding, for a given galaxy, such a spatial rotation field and mass distribution for an n-body system – a computer model of this galaxy – which would be able to reconstruct at least the observed Doppler image of the galaxy. Reconstruction of a published rotation curve only, is not suf-

ficient, because different groups obtain for the same galaxy different rotation curves depending on the details of their procedures applied for processing Doppler images. Such images are also often nonsymmetric, unlike simple galaxy models, and lot of information about galaxy kinematics they convey is missing by the fact that the images are by construction a sort of projection of a three dimensional rotational field onto the observation plane.

1.3 Luminosity

M/L ratio is important for studying the formation and evolution of galaxies, thus already from this standpoint it is rather strange to assume it is constant from the start, while it would be more desirable to determine it as a function of radius. Unfortunately, the theoretical M/L for an individual galaxy is not well known even when assumed constant. Its derived value depends strongly on the assumed mass function for the stellar population, on the star formation history, on metallicity and depends also on extinction. It is also affected by the assumed internal extinction law, by the errors in luminosity measurements, and by the distance dimming (de Blok et al. 2001). This ratio is generally assumed constant separately for the bulge and for the disk. Accordingly, one tacitly assumes that the contribution of bulge and that of disk to the rotation curve should be controlled only by the light distribution via a single number found by best fits as if brightness profiles were to trace the overall features of rotation curves. However, the assumption of constant M/L ratio may not be correct, since it requires that extinction and population gradients across the luminous disk should be small (Palunas & Williams 2000). Constant M/L ratio seems also improbable since it would imply that composition of a galaxy is homogenous.

Overestimating the importance of constant M/L may change considerably and in uncontrolled way the contribution of bulge and that of disk to the overall rotation, the more that optical rotation curves are not featureless, while luminosity is usually fitted by simple laws (exponential, *etc*). In particular, this may lead to errors in reconstruction of the relation between mass density and rotation of the disk component, since rotation of a disk is a nonlocal functional of the density profile, sensitive to the detailed local structure of the profile. This in turn may lead to wrong estimates of mass profiles of other mass components. To give only an example, many spirals simply do not have exponential disks or any other given by a simple analytical law, and assuming otherwise, results in large discrepancies in disk scale lengths published by different authors for the same galaxies (Palunas & Williams 2000). Closely related to the non-uniqueness of the estimates of mass profiles is the so called disk-halo degeneracy – the uncertainty in the relative contributions of different galaxy components to the overall rotation; for unambiguous disk-halo-bulge decomposition one requires additional constraints on mass distribution, in particular, such as the mass-to-light ratios or the assumed mass profiles and shapes of dark haloes. In finding such constraints may also be helpful measurements of gravitational lensing in spiral galaxies (Maller et al. 2000).

Usual rotation-curve fitting methods have too many free parameters and are not unique, require additional assumptions, sometimes quite arbitrary. For example, de Blok et al.

(2001) present disk-halo decompositions with several different assumptions for stellar M/L ratio for the same galaxy, or assume that only the minimum disk is present and that the observed rotation curve is due entirely to dark matter, or assume that stellar M/L ratio is zero (!) and take into account contribution of the atomic gas (H I and He). In contrast, our approach does not have the arbitrariness problem, since we predict, not assume mass to light ratios.

As pointed out by Ciardullo, Durrell, et al (2004), there is only limited evidence for constant M/L . What's more, the results of Herrmann and Ciardullo (2005) for M33 suggest that this assumption is not valid at all, M/L ratio of the galaxy's disk increases approximately 5 times over 6 scale lengths of the inner disk. Their findings also suggest that there is very little dark matter in M33's central regions.

2 MOTIVATION FOR THE USE OF A GLOBAL DISK MODEL

As we have seen above, the case for dark matter inside galaxies is very weak and the amount of ascertained mass of dark matter is strongly model-dependent. It should be stressed that the models are constructed in the same framework of Newtonian gravitation. There is increasing number of papers where more and more complex models are considered compared with previous simple models of galaxies. They account for the observed rotation of internal regions of flattened galaxies without dark matter, and some of them, based on maximal disk hypothesis, account for global rotation also without dark matter. The use of models with extended disk finds additional support from the fact that dark haloes may be very flattened rather than spherical. For flattened systems, however, gravitation gets more complicated and new effects difficult to tackle with, such as influence of external masses on internal orbits, must be taken into account and which would be absent if external masses were distributed spherically symmetric. Also stellar mass-to-light ratio need not be constant. Assuming otherwise, galaxy mass models become very stiff leading, for example, to disk-halo conspiracy. As we saw, this also leads to discrepancies between predictions.

As concerns the presence of dark matter in spiral galaxies, we therefore prefer a more cautious approach than just to assume the presence of the (unobserved) dark matter halo from the beginning. We propose to admit dark matter, only if models that composed of baryonic matter distributed in a flattened disk (of stars and gas) and of more spherical bulge and stellar halo, fail to properly account for the dynamics of the disk measured through rotation curves. Such an approach, we believe, could significantly reduce the undesirable model-dependence of the amount of CDM in spiral galaxies (Jalocha et al. 2008; Bratek et al. 2008).

Most models of galaxy formation predict that M/L should be a declining function of distance, e.g. the inner regions of a galaxy form first and contain older, higher M/L ratio star populations (Ciardullo, Durrell, et al 2004). As follows from the previous section it is not correct to assume that the ratio is constant throughout galaxies. This is the reason why we prefer to determine mass distribution in disk based on measurements other than brightness profiles first, and only then determine mass-to-light ratios as a result of

comparison of the obtained mass profile with the measured luminosity profile. In several galaxies we have studied so far using our approach we observe that this ratio indeed decreases while density approaches that of hydrogen and helium known from measurements. This also suggest that dark matter is not needed for some galaxies even in the outermost regions.

Conceptually simple, direct method for determining the gross mass distribution in galaxies, is rotation curve inversion. Maximal disk model is an example. Provided rotation curve is known globally, the corresponding mass distribution can be found via single integral functional. The method is independent of any conventional assumptions concerning decomposition of a galaxy into various idealized subsystems with their unknown (usually correlated with each other) parameters determinable from best fits. Unfortunately, direct rotation curve inversion is not unambiguous if mass distribution does not have special symmetries, e.g. spherical symmetry.

Various ambiguities pertinent to gravitation of flattened systems, easily hidden by the use of a particular model for the dark mass (Sackett 1996), may influence the actual importance of different galaxy components. This fact seems not much appreciated. In addition, there is also ambiguity in mass-to-light ratios within optical disks, since extended rotation curves alone give no indication of where the luminous disk could end (Bahcall & Casertano 1985). Consequently, as noted in (Palunas & Williams 2000), there are mass models which fit rotation curves within the optical disk both with and without significant dark component (van Albada & Sancisi 1986; Lake & Feinswog 1989). What's more, rotation of a galaxy considered dark matter dominated in one model can be devoid of dark matter in another model (Jalocha et al. 2008).

For a flattened mass distribution, local density is a non-local functional of the rotation field. And vice versa, velocity on a given orbit is affected both by gravitation of masses external and internal to that orbit. For that example, it would be impossible to reconstruct dynamical mass function from rotation of luminous matter moving in a potential well of unseen external very flattened halo of dark matter. Such dark haloes considerably flattened toward the stellar plane, resembling a disk more closely than a sphere (Sackett 1996), with axis ratios between 0.1 and 0.3 where suggested by Sackett et al. (1994) and Olling (1996). Bratek et al. (2008) analyzed errors resulting from extrapolation of the rotation curve beyond the last measured point and gave a criterion for their estimation in global disk model. These errors are the fundamental obstacle in using disk models. The reconstructed mass density is viable only out to a distance of 60% of the radius of the last measurement point. The less serious is sensitiveness of this method to noise criticized in (Binney & Tremaine 1987). However, as Sackett (1997) illustrates, this leads only to 10 – 20% uncertainty. Therefore one needs a method to minimize such errors.

Once one accepts that very flattened haloes of unseen matter can exist then one may lose any predictability. For example, since dark haloes are not seen, one may assume anything about them, say, that outside the last measurement point there is present a flattened dark matter ring of a given arbitrarily large mass and given shape, encircling the galaxy.

To balance its influence on the luminous matter, one needs correspondingly large amount of mass inside galaxy. This in turn would made impossible even to determine the mass-to-light ratio of the luminous matter since the ratio would depend on the unknown mass of the flattened dark matter halo.

There is a simple criterion for deciding, whether the mass distribution at larger radii in some galaxy is flattened rather than spherical. In the presence of spherical dark matter halo, the non-spherical component of mass distribution should be negligible. Then, only the radial component of the gravitational force is important. In this case Keplerian mass function $G^{-1}rv_c^2(r)$ defined for rotation curve $v_c(r)$ must be a nondecreasing function of radius

$$\frac{d}{dr} \left[\frac{rv_c^2(r)}{G} \right] \geq 0. \quad (1)$$

In deriving this inequality, we assumed circular orbits of matter in the vicinity of the galactic plane, which is customary in modeling of rotation curves. For spherical systems, the Keplerian mass function is identical with the true mass function. Thus, if sphericity condition (1) is not satisfied at larger radii, the true mass distribution cannot be spherical, and spherical CDM halo is thereby excluded (as far as the usual assumptions in galaxy modeling are employed), or other forms of dark matter are required forming flattened structures.

We can use the global disk model especially for galaxies with rotation curves breaking the sphericity condition at larger radii. Note, that the distribution of internal masses is not much important for the overall gravitation at larger radii (contribution from higher gravitational multipoles of internal masses decreases quickly with radius), therefore, the disk model can be used also for the inner parts, in spite of the fact that the center may be rather spheroidal since, after de-projection of a part of the disk density onto a spheroidal component, this does not change the central mass significantly – this is an approximation, a standard procedure in the maximal disk model. Since rotation curves of disks satisfying the sphericity condition are also possible (Bratek et al. 2008), one can try to apply the disk model also to spiral galaxies of which rotation curves do not violate this condition.

2.1 The model

The disk model assumes that whole matter in a galaxy is distributed in the vicinity of the galactic equatorial plane. The model also assumes that orbits of stars are circular and that dispersion of velocities is negligible. The other simplifying assumption is that whole system is axisymmetric. The relation between surface mass density $\sigma(r)$ and velocity is then given by

$$v^2(r) = 4Gr V.p. \left(\int_0^r \sigma(\chi) \frac{\chi E\left(\frac{\chi}{r}\right)}{r^2 - \chi^2} d\chi \dots \right. \quad (2)$$

$$\left. \dots - \int_r^\infty \sigma(\chi) \left[\frac{\chi^2 E\left(\frac{r}{\chi}\right)}{r(\chi^2 - r^2)} - \frac{K\left(\frac{r}{\chi}\right)}{r} \right] d\chi \right),$$

where K and E are elliptic functions of the first and second kind (Bratek et al. 2008). This integral is understood in the principal value sense (*V.p.*), since both summands are divergent, thus the integral must be skilfully calculated.

In practice, however, it suffices to assume that matter is located close to the galactic plane only in the galaxy outskirts where the external orbits are stabilized by gravitational potential of the galactic interior (this contribution to gravitational potential is almost spherical at large radii), while in the vicinity of galactic interior the density can be treated as an effective density understood rather as a column density of masses projected onto the galactic plane – a substitution for otherwise spherical and stable bulges *etc* in internal regions. Interpreted in this way, this approximated cold disc model of an otherwise stable spiral galaxy also avoids, or at least weakens, the stability counterarguments often raised against global disk models (see also the discussion in section 1.2).

It is seen from equation (2) that for a flattened mass distribution of which the disk model is a limiting approximation, rotational velocity of matter on a given orbit is dependent both of masses distributed interior and exterior to that orbit. The inverse relation is a functional of whole rotation curve, and it reads (Jalocha et al. 2008; Bratek et al. 2008)

$$\sigma(r) = \frac{1}{\pi^2 G} V.p. \left[\int_0^r v^2(\chi) \left(\frac{K\left(\frac{\chi}{r}\right)}{r\chi} - \frac{r}{\chi} \frac{E\left(\frac{\chi}{r}\right)}{r^2 - \chi^2} \right) d\chi \dots \right. \quad (3)$$

$$\left. \dots + \int_r^\infty v^2(\chi) \frac{E\left(\frac{r}{\chi}\right)}{\chi^2 - r^2} d\chi \right]$$

In the literature of this subject, for some mysterious reason, there is always used an (equivalent) expression containing derivatives of velocities which are hardly measurable with satisfactory accuracy, e.g. (Binney & Tremaine 1987).

The crucial point now is that a rotation curve is never known globally, thus the mass distribution even in the region where rotation is known, cannot be determined from rotation only. This is the manifestation of "non-locality" of gravity of flattened systems – one cannot determine their mass functions from the observed rotation and, therefore, additional constraints on this mass distribution is required. This is particularly important in the regions where measurements of rotation end, while the analogous error is negligible in the internal galaxy regions. This property is fatal for reconstruction of mass distribution inside a galaxy when outside the luminous part of this galaxy there is present an external ring-like halo composed of dark matter; if it were spherical there would be no problem at all.

Having realized the unpleasant properties of flattened galaxies we devised in (Jalocha et al. 2008) a method which may help to minimize this uncertainty or remove it. One does not have to assume anything beyond the last measured point, but simply use the available data of mass distribution as a lower bound for the amount of matter in regions where rotation for some reasons could not be determined, but the amount of gas is still measurable. Our goal is to examine how good is our model in reproducing the mass distribution in a sample of spiral galaxies.

The thin disk model's role is to find such a global $\sigma(r)$,

that the corresponding velocity of rotation $v(r)$ defined by integral (2), would best overlap with the observed rotation curve $v_c(r)$. This task, however, is not unique, since for every R , there is an infinite set of functions $\sigma(r)$ that give rise to the same $v(r)$ for $r < R$, even though the $\sigma(r)$'s may significantly differ from one another for $r > R$ and have different total masses. To minimize this non-uniqueness, we look for mass distribution in flattened galaxies by iterations. A particular realization of such iteration proposed in (Jalocha et al. 2008) assumes that mass distribution of gas in a given galaxy is known for radii greater than the range of the rotation curve.

We bring to the reader's attention the fact, that mass distribution found in (Jalocha et al. 2008) for galaxy NGC4736 using this method, is consistent with all measurements, and this was possible without dark matter. This result is interesting, since another model of this galaxy (Kent 1987) predicts almost 70% abundance of dark matter and does not well explain its rotation at large radii. It is therefore natural to examine whether this galaxy is exceptional or, maybe, if there are other spiral galaxies with lower abundance of dark matter.

3 RESULTS

Below, we present results for several galaxies obtained with the use of global disk model. We applied the iteration method analogous to that introduced by Jalocha et al. (2008). We are aware this is not a representative sample of galaxies. However, we think, that even when rotation of only a single galaxy could be explained without dark matter, this would be interesting as such and worthy to be presented.

Initially, we planned to examine only galaxies with rotation curves breaking the condition of spherical mass distribution, because such galaxies are presumably flattened and, therefore, natural candidates to which global disk model is applicable. However, in order to see whether the model produces viable results for other galaxies, we included also galaxies NGC 6946 and UGC 6446 for which we also found good and complete data.

As concerns the error analysis, our method by construction exactly reproduces rotation curves, thus also fits rotation measurements within error bars. This is the reason why we show rotation curves without error bars.

3.1 NGC 4736

Here, we once again consider galaxy NGC 4736, which earlier was studied in (Jalocha et al. 2008). This time, however, we use another, newer rotation curve published by de Blok et al. (2008). In figure 1, this new curve is compared with that obtained by Sofue (1997) we used last time. Both the curves are shown assuming distance 4.7Mpc different from the previous 5.1Mpc. Although both curves do not overlap we come to comparable conclusions concerning the amount of non-baryonic dark matter. NGC 4736 still remains a galaxy in which the halo is not necessarily required. Surface density obtained in the global disk model smoothly approaches that of gas in the outskirts of the galaxy, we obtain also low M/L ratios (1.28 for I-band, 1.44 for V-band,

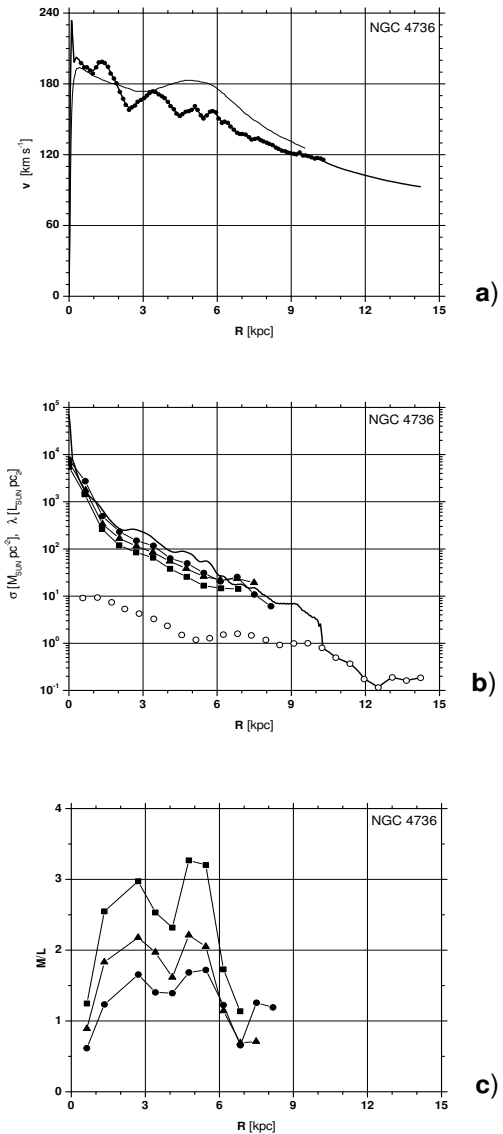


Figure 1. Results obtained in the global disk model for galaxy NGC 4736 at $D = 4.7 \text{ Mpc}$. **a)** Rotation curve: THINGS measurements [solid circles] (de Blok et al. 2008), the model [solid line], and comparison with high resolution rotation curve taken from (Sofue 1997). **b)** model global surface mass density [solid line], surface mass density of HI+He [open circles]; and surface brightness: B-band [solid squares], V-band [solid triangles], I-band [solid circles] (Munoz et al. 1989); **c)** mass-to-light ratio profile without inclusion of HI+He.

and 2.01 for B-band). In addition, the local M/L ratio decreases with radius for the largest radii. Therefore, despite the new rotation curve does not break sphericity condition so strongly unlike that by Sofue (1997), we still consider this galaxy as containing no dark matter or that abundance of dark matter is at most very low.

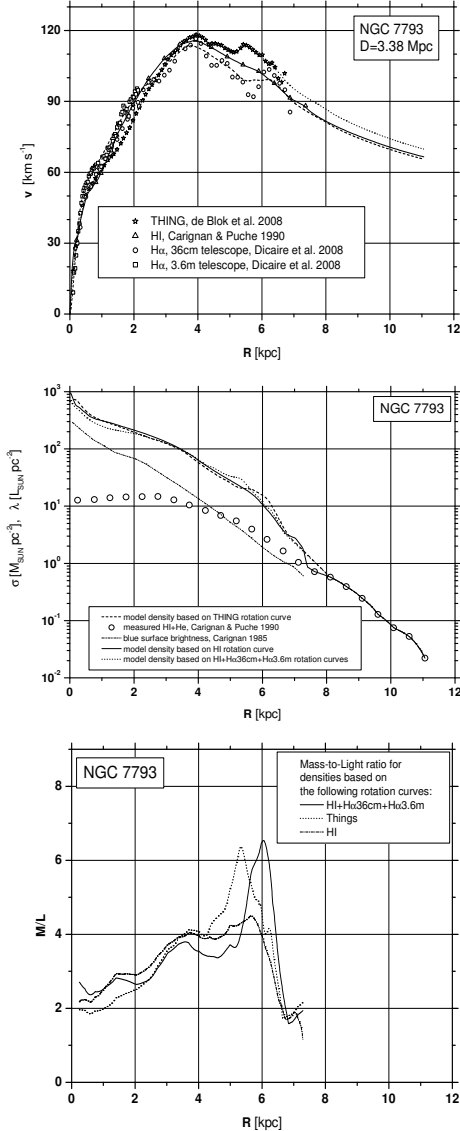


Figure 2. *Upper:* Comparison of the NGC 7793 rotation curves obtained in THINGS (de Blok et al. 2008), (Carignan & Puche 1990) and (Dicaire et al. 2008). Symbols stand for measurements and lines for model fits. *Middle:* surface mass densities obtained in global disk model compared with surface mass densities of neutral hydrogen and helium and with surface brightness in the B-band. *Bottom:* Mass-to-light ratios derived based on mass densities obtained for different rotation curves.

3.2 NGC 7793

We take rotation curves published by a) Carignan & Puche (1990), b) de Blok et al. (2008) and c) three rotation curves published by Dicaire et al. (2008). The latter three curves were merged such to obtain a single rotation curve. In effect we have three distinct rotation curves for the same galaxy. As seen in figure 2 the curves are similar which shows rotation measurements for this galaxy are trustwor-

thy. The corresponding masses are also close to each other, $1 - 1.02 \times 10^{10} M_{\odot}$. Surface mass density from the global disk model smoothly converges to that of gas at the outskirts of the galaxy for all three cases. We obtain also low values for M/L ratios (2.93 – 3 in B-band). We stress that the attempt of de Blok et al. (2008) to account for the rotation with spherical CDM halo, gives very poor results. By giving up the halo, we obtain surface mass density in agreement with the amount of gas, and low M/L ratios. Therefore, we think it is reasonable to assume that the galaxy does not have any dark matter halo.

3.3 NGC 1365

Now we consider two rotation curves (Sofue 1997) and (Zanmar et al. 2008) the latter suitably smoothed out in a way to conform with data points and error bars. We have noticed that surface mass density corresponding to the rotation curve has approximately exponential falloff (see figure 3). Therefore, for a while one may assume that surface mass density is a superposition of two exponential profiles, in order to check whether such mass distribution accounts for the observed rotation. We find, this is indeed the case, however the mass density becomes larger than that of gas (although quickly falling off). On the other hand, the surface density obtained with our method converges smoothly to that of gas already in a place where galaxy is still bright in B-band. This can be interpreted as that apart from HI and He there must be present other luminous stuff. In general, this is very interesting, that for this galaxy already the doubly-exponential disk alone suffices to account for the rotation curve of (Zanmar et al. 2008). Irrespectively of which rotation curve is considered, the obtained mass-to-light ratio is low (2.91 to 3.15 in B-band depending on which rotation curve and method is used). This ratio falls-off with increasing radius in every case. As so, galaxy NGC 1365 can be modeled without non-baryonic dark matter.

3.4 NGC 6946

We take three rotation curves published by Sanders (1996), by Boomsma et al. (2008) and de Blok et al. (2008) (the latter were smoothed out). As is seen, the published rotation data differ significantly from each other, particularly rotation curve of de Blok et al. (2008) exceeds much from the other. By using global disk model we obtained surface densities which smoothly converges to that of gas. Mass-to-light ratio is low based on rotation by Sanders (3.49 in B-band), but it becomes twice greater for rotation curve of THINGS (6.15) M/L ratio for THINGS grows slightly with radius, for the other two curves it is approximately constant. Summing up, based on available rotation data it is difficult to uniquely decide about presence of non-baryonic dark matter in this galaxy. The result depends of which rotation curve is used.

Using our modeling method we can explain all these rotation curves. It is worth of pointing out that the obtained surface densities smoothly converge to that of gas. However, based on THINGS rotation curve, we obtain M/L ratio much greater than for NGC 4736, NGC 1365, NGC 7793, similarly, for the other two rotation curves.

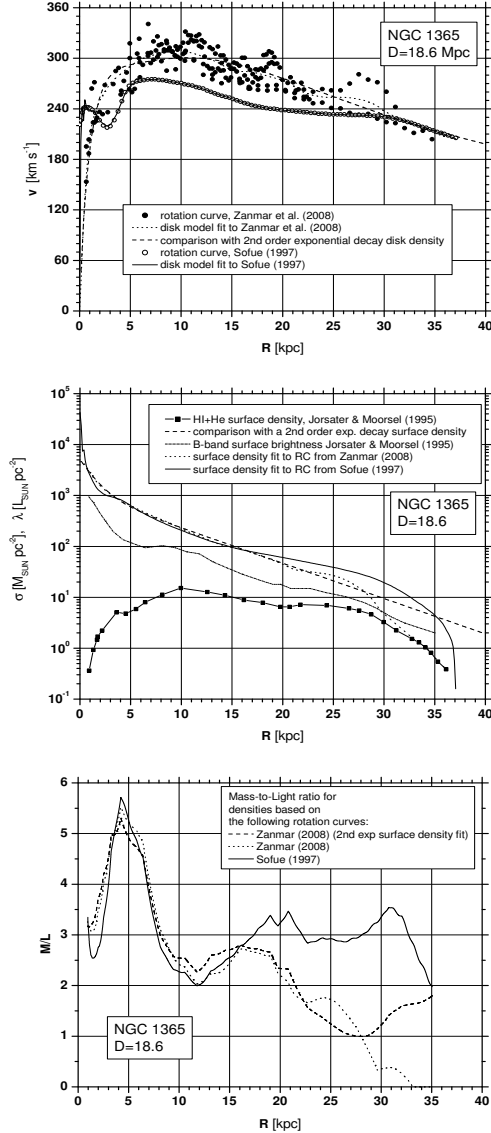


Figure 3. *Upper:* Comparison of the NGC 1365 rotation curves obtained in (Zanmar et al. 2008) and (Sofue 1997). Symbols stand for measurements and lines for model fits. *Middle:* surface mass densities obtained in global disk model compared with a 2nd order exponential decay fit and with surface mass densities of neutral hydrogen and helium and surface brightness in the B-band. *Bottom:* Mass-to-light ratios derived based on mass densities obtained different rotation curves.

3.5 UGC 6446

The M/L profile of galaxy UGC 6446 grows with radius attaining values much higher than for the other examined galaxies, even after subtraction of HI and He. Such a singular behavior of the M/L profile is expected at the edge of luminous disk, where the luminosity per unit of surface tends to zero, $\lambda(r) \rightarrow 0$, while surface mass density remains finite. Then, the local M/L ratio becomes singular,

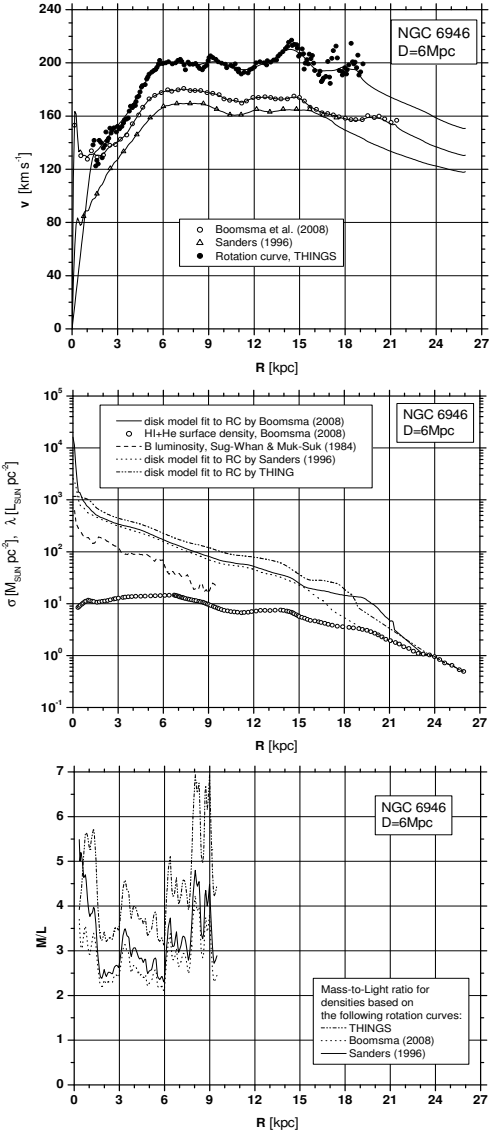


Figure 4. *Upper:* Comparison of the NGC 6946 rotation curves obtained in (Boomsma et al. 2008), (Sanders 1996) and THINGS (de Blok et al. 2008). Symbols stand for measurements and lines for model fits. *Middle:* surface mass densities obtained in global disk model compared with surface mass densities of neutral hydrogen and helium and surface brightness in the B-band. *Bottom:* Mass-to-light ratios derived based on mass densities obtained different rotation curves.

$\sigma(r)/\lambda(r) \rightarrow \infty$. After subtraction of mass contribution of gas, the increasing M/L ratio indicates, that the stars at the outskirts of luminous disk, contain possible admixture of died stars, "black" dwarfs, neutron stars, or stars of too low a mass for nuclear burning (so called brown dwarfs). It is important to note, that the mass density of such a died star component needs only to be residual, such that total mass density is infinitesimally bigger than the mass density of shining stars, $\sigma_* + \epsilon$ for the M/L ratio significantly in-

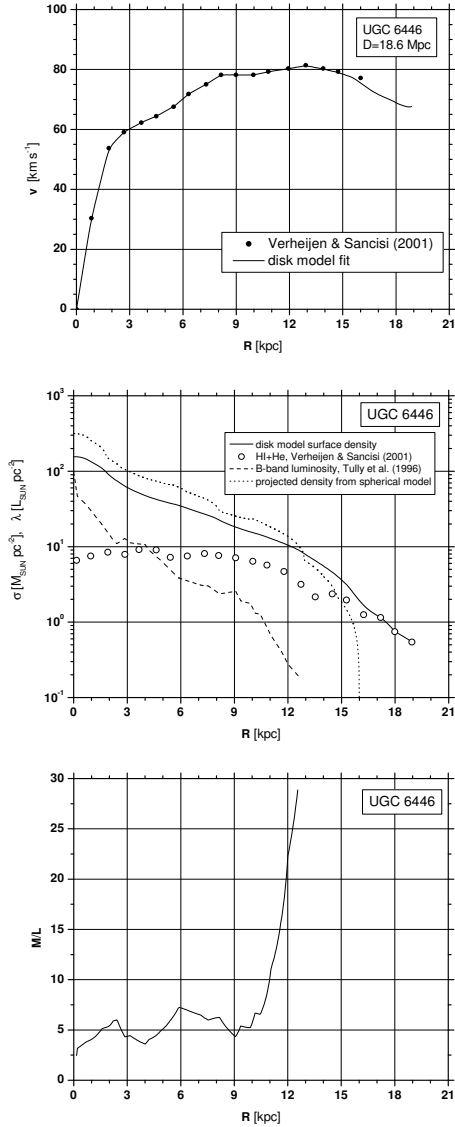


Figure 5. *Upper:* Comparison of the NGC 6446 rotation curve obtained in (Verheijen & Sancisi 2001). Symbols stand for measurements and lines for model fits. *Middle:* surface mass density obtained in global disk model compared with that of obtained in spherical mass model, with that of neutral hydrogen and helium and with surface brightness in the B-band. *Bottom:* Mass-to-light ratio derived based on mass density obtained different rotation curves.

creases at the edge: $(\sigma_* + \epsilon)/\lambda \rightarrow \epsilon/\lambda$ when both $\sigma_*(r) \rightarrow 0$ and $\lambda(r) \rightarrow 0$. This very small mass density, ϵ , of died stars does not show up anywhere except at the very edge where it is strongly enhanced by small $\lambda(r)$. Contribution of died stars to the mass of galaxy is thus negligible.

The following thought experiment provides a strong argument against the existence of a spherical halo around galaxy UGC 6446. Suppose for a while that this galaxy is dominated by such a halo. Then, the mass distribu-

tion could be considered spherical with good accuracy, thus the corresponding mass function could be calculated from $M(r) = \frac{rv^2(r)}{G}$ (and this can be done, since the rotation curve of this galaxy satisfies the sphericity condition). Then, the corresponding column mass density σ_{col} can be found by projecting the volume density onto the galactic plane by using the formula

$$\mu(r) := \frac{rv^2(r)}{G}; \quad \sigma_{col}(\rho) = \frac{\mu(R) - \mu(\rho)}{2\pi R \sqrt{R^2 - \rho^2}} + \dots \quad (4)$$

$$+ \lim_{\epsilon \rightarrow 0} \frac{1}{2\pi} \int_{\sqrt{\epsilon^2 + \rho^2}}^R \frac{2r^2 - \rho^2}{r^2(r^2 - \rho^2)^{3/2}} \left[\mu(r) - \mu\left(\sqrt{\epsilon^2 + \rho^2}\right) \right],$$

where R is the distance of the outermost point on the rotation curve. Note, that our formula is very advantageous from the practical point of view, contrary to the (equivalent) handbook formula, as it does not require derivatives of velocities, which are measured with very poor accuracy. We compare the column density σ_{col} with total surface mass density obtained from the disk model and with its component surface mass density of gas (HI+He). The result is shown in figure 5. We see that close to the "edge", σ_{col} is lower than the amount of gas! We stress that the two surface mass densities seen in figure 5 correspond to two extreme idealizations – in the first the galaxy is approximated by a thin disc, in the other, by a spherically symmetric mass distribution. The realistic mass distribution must be somewhere in between, but, as is seen, closer to that in disk model. In view of these observations, we classify UGC 6446 as a galaxy without massive spheroidal dark halo.

4 SUMMARY

We considered 5 galaxies, among which one was reexamined with a new rotation curve. Our aim was to answer the question if the galaxies could be satisfactorily modeled without non-baryonic dark matter. In this regard we check whether the following conditions are met: the found mass distribution should account for the measured rotation curve, it should approximately converge toward that observed for gas (HI + He) in the outer parts of galaxies (as this is the lower bound for the amount of luminous matter), the mass-to-light ratio should be low and its profile should be a declining function of radius close to the galaxy "edge".

We find that these conditions are satisfied by our results for galaxies NGC 4736, NGC 7793 and NGC 1365 for all rotation curves. It is reasonable to state that these are galaxies with flattened, disk-like mass distributions, for which the disk model approximation works quite well. The case of galaxy NGC 6946 is not so clear. There are significant discrepancies between published rotation curves, and consequently, in the obtained M/L ratios. Galaxy UGC 6446 satisfies some of our criteria without non-baryonic dark matter, however its M/L ratio is highest and, in addition, it grows dramatically toward galaxy "edge". In another test we assumed that the galaxy is dominated by a spherical dark matter halo and, using a spherical model, obtained the resulting bulk mass density. This density, accounting for the galaxy rotation, is in this case lower than that of gas, which can be considered as a strong argument for absence of spherical non-baryonic dark halo. This argument does not exclude the

presence of a non-spherical halo. However, the mass needed in the galaxy outskirts to account for the rotation in the disk model is practically that of gas. It would be therefore worthwhile to consider other than CDM causes of the M/L ratio growth.

We have also signalized the controversial issue, to what extent the very simplified galaxy models like the ones commonly used by astronomers or like that used in the current paper, can be treated as models of realistic galaxies. In particular, whether stability arguments can be applied to favourize or reject a particular galaxy model, since the role of these models, which serve only as an idealized approximation incorporating many assumptions, is to estimate different global characteristics of galaxies like total mass, the amount of gas, global mass-to-light ratio, dark-to-luminous mass ratio *etc.*, not to reconstruct exact spatial mass distribution, the more that many effects like the influence of global magnetic fields in a galaxy on the rotation of gas in the galaxy outskirts, are normally not taken into account, and these fields may be important for accounting for flat rotation curves (Battaner, Garrido, Membrado & Florido 1992).

Global disk model is a natural approximation for flat-tened galaxies such as spiral ones, as far as it is not assumed in advance, sometimes by force, that each spiral galaxy must necessarily be immersed in an invisible halo of dark matter. Moreover, we are able to find such mass distributions which perfectly account for rotation curves, while in models assuming massive halo these rotation curves often cannot be satisfactorily reconstructed, eg. in the case of NGC 7793 in (de Blok et al. 2008), or the dark halo found is too weak to account for stability criteria (NGC 4736 in (de Blok et al. 2008)) (whereas, historically, the role of such haloes was not only to explain flat rotation, but also to stabilize disks). Although our model is very simple, similarly as other galaxy models customarily used by astronomers, it is better by that it does not assume constant mass-to-light ratios but it predicts the ratio as a resultant mass-to-light profile. Our results pertain only to the particular galaxies examined by us so far. From the fact that there is no need of non-baryonic dark matter in galaxies NGC 4736 or NGC 7793, one cannot conclude that one does not need it in other spiral galaxies. To claim otherwise, further studies are needed. Nonetheless, we find interesting the existence of spiral galaxies without non-baryonic dark matter or galaxies with small abundance of such matter.

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name	NGC 7793			NGC 1365			NGC 6946		
incl. angle [deg]	53.7 ^A			46 ^B			30 ^B		
distance [Mpc]	3.38 ^A			18.6 ^L			6.0 ^M		
morphological type	SAd ^A			SBb ^B			SAbc ^B		
L_B [$10^{10} L_\odot$]	0.3 ^K			9.81 ^E			1.64 ^H		
M_{H+He} [$10^{10} M_\odot$]	0.12 ^A			2.36 ^E			0.97 ^M		
M_{H_2} [$10^{10} M_\odot$]	-			-			0.45 ^J		
rotation curve	HI ^A	$HI + H_\alpha$ ^O	Thing ^P	B	L	L Π_{exp}	R	M	P
M_{Gal} [$10^{10} M_\odot$]	1	1.02	1	33.4	30.9	31.9 $R = 40kpc$	7.14	8.55	11.5
breaking sphericity test	Y	Y	Y	Y	Y	N	N	Y	Y
M_{Gal}/L_B	3.33	3.4	3.33	3.4	3.15	3.25	4.35	5.21	7.01
$\frac{M_{Gal}-M_{gas}}{L_B}$	2.93	3	2.93	3.16	2.91	3.01	3.49	4.35	6.15

name	NGC 4736	UGC 6446
incl. angle [deg]	35 ^B	44 ^D
distance [Mpc]	4.7 ^P	18.6 ^G
morphological type	Sab ^B	Sd ^G
L_B [$10^{10} L_\odot$]	1.3 ^S	0.218 ^J
M_{H+He} [$10^{10} M_\odot$]	0.067 ^T	0.434 ^G
M_{H_2} [$10^{10} M_\odot$]	-	-
rotation curve	Thing	G
M_{Gal} [$10^{10} M_\odot$]	2.68	1.5
breaking sphericity test	Y	N
M_{Gal}/L_B	2.06	6.88
$\frac{M_{Gal}-M_{gas}}{L_B}$	2.01	4.89

Table 1. Results obtained in the framework of global disk model for various spiral galaxies and references to the used measurement data: ^A – (Carignan & Puche 1990), ^B – (Sofue 1997), ^D – (HyperLeda), ^E – (Jorsater & Moorsel 1995), ^F – (Hoekstra et al. 2001), ^G – (Verheijen & Sancisi 2001), ^H – (Sug-Whan & Muk-Suk 1984), ^I – (Young & Scoville 1991), ^J – (Tully et al. 1996), ^K – (Carignan 1985), ^L – (Zanmar et al. 2008), ^M – (Boomsma et al. 2008), ^O – (Dicaire et al. 2008), ^P – (de Blok et al. 2008), ^R – (Sanders 1996), ^S – (Munoz et al. 1989), ^T – (Mulder & van Driel 1993)

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